A Letter of Dr. John Wallis *to* Samuel Pepys *Eſquire, relating to ſome ſuppoſed* Imperfections *in an* Organ.

Mr. *Harris* an Organ-maker (whom I find, by the little diſcourſe I had with him, to be very well skilled in his profeſſion) was lately with me, as by direction from you, to ask my opinion about perfecting an Organ, in a point wherein he thinks it yet Imperfect.

'Tis an honour you pleaſe to put upon me, to think my opinion conſiderable in a thing wherein I am ſo little acquainted as that of an Organ.

I do not pretend to be perfectly acquainted with the Structure of an Organ, its ſeveral Parts, and the Incidens thereunto; Having never had Occaſion and Opportunity to inform my ſelf particularly therein. And, for the ſame reaſon, many of the Words, Phraſes, Forms of Speech, and Terms of Art, which are familiar to Organiſts and Organ-makers, are not ſo to me. Which therefore I ſhall wave; (For till we perfectly underſtand one anothers Language, it is not eaſy to ſpeak intelligibly;) and apply my ſelf directly to what is particularly propoſed.

This (I take it) is evident; That each *Pipe* in the Organ is intended to expreſs a diſtinct *Sound* at ſuch a *Pitch*; That is, in ſuch a determinate Degree of *Gravity* or *Acuteneſs*; or (as it is now called) *Flatneſs* or *Sharpneſs*. And the *Relative* or *Comparative* Conſideration of Two (or more) ſuch Sounds or Degrees of Flatneſs and Sharpneſs, is the ground of (what we call) *Concord* and *Diſcord*; that is, a Soft, or Harſh, coincidence.

Now, concerning this, there were amongſt the Ancient Greeks, Two (the moſt conſiderable) *Sects* of Muſicians: the *Ariſtoxenians*, and the *Pythagorians*.

They both agreed thus far; That *Dia-teſſaron* and *Dia-pente*, do together make-up *Dia-paſon* ; that is (as we now ſpeak) a *Fourth* and *Fifth* do together make an *Eighth* or *Octave*: And, the *Difference* of thoſe two (of a Fourth and Fifth) they agreed to call a *Tone* ; which we now call a *Whole note*.

Such is that, (in our preſent Muſick,) of *La Mi* , (or as it was wont to be called, *Re Mi.*) For *La fa ſol la*, or *Mi fa ſol la*, is a perfect *Fourth* : And *La fa ſol la mi*, or *La mi fa ſol la*, is a perfect *Fifth* : The *Difference* of which, is *La mi*. Which is, what the Greeks call, the *Diazeuctick Tone* ; which doth *Dis-join* two *Fourths* (on each ſide of it ;) and, being added to either of them, doth make a *Fifth*. Which was, in their Muſick, that from *Meſe* to *Parameſe* ; that is in our Muſick, from A to B : ſuppoſing *Mi* to ſtand in B *fa* b *mi*, which is accounted its *Natural* poſition.

Now, in order to this, *Ariſtoxenus* and his Followers, did take, that of a *Fourth*, as a *Known Interval*, by the judgement of the Ear ; and, that of a *Fifth*, likewiſe ; And conſequently, that of an *Octave*, as the *Aggregate* of both ; and that of a *Tone*, as the *Difference* of thoſe Two.

And this of a *Tone* (as a *known Interval*) they took as a *common Meaſure*, by which they did eſtimate other Intervals. And accordingly they accounted a *Fourth* to contain *Two Tones and an half*; a *Fifth* to contain *Three Tones and an half*; and conſequently an *Eighth* to contain *Six Tones*, or *Five Tones and two Half-tones*. And it is very near the matter, though not exactly ſo.

And at this rate we commonly ſpeak at this day ; ſuppoſing an *Octave* to conſiſt of *Twelve Hemitones*, or Half-notes. (Meaning thereby, ſomewhat near ſo many half-notes :) But, when we would ſpeak more Nicely, we do not take thoſe ſuppoſed *Half-notes* to be exactly *Equal*, or each of them juſt the *Half* of a *Full-note*, ſuch as is that of *La-mi*.

Pythagoras and thoſe who follow him, not taking the Ear alone to be a competent Judge in a caſe ſo nice ; choſe to diſtinguiſh theſe, not by *Intervals*, but by *Proportions*. And accordingly they accounted that of an *Octave*, to be, when the degree of Gravity or Acuteneſs of the one Sound to that of the other, is *Double*, or as 2 to 1 ; that of a *Fifth*, when it is *Seſqui-alter*, or as 3 to 2 ; that of a *Fourth* when *Seſqui-tertian*, or as 4 to 3. Accounting That, the Sweeteſt proportion, which is expreſſed in the Smalleſt Numbers ; and therefore (next to the *Uniſone*) that of an *Octave*, 2 to 1; then that of *a Fifth*, 3 to 2; and then that of a *Fourth*, 4 to 3.

And thus, that of a *Fourth* and *Fifth*, do together make an *Eighth*; For $\frac{4}{3} \times \frac{3}{2} = \frac{4}{2} = \frac{2}{1} = 2$. That is, *four thirds* of *three halves*, is the ſame as *four halves*, that is *Two*. Or (in other words to the ſame ſenſe) the proportion of 4 to 3, compounded with that of 3 to 2, is the ſame with that of 4 to 2, or 2 to 1. And, conſequently, the *Difference* of thoſe Two, which is that of a *Tone* or *Full-Note*, is that of 9 to 8. For $\frac{4}{3})\frac{3}{2}(\frac{9}{8}$; that is, *three halves* divided by *four thirds*, is *nine eights*; or, if out of the proportion of 3 to 2, we take that of 4 to 3; the Reſult is that of 9 to 8.

Now, according to this Computation, it is manifeſt, That an *Octave* is ſomewhat leſs than *Six Full-notes*. For (as was firſt demonſtrated by *Euclide*, and ſince by others) the Proportion of 9 to 8, being ſix times compounded, is ſomewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Caſe; they allowed (indiſputably) to that of the *Dia-zeuctick Tone* (*La mi*,) the full proportion of 9 to 8; as a thing not to be altered; being the Difference of *Dia-pente* and *Dia-teſſaron*, or the *Fifth* and *Fourth*.

All the Difficulty, was, How the remaining *Fourth* (*Mi fa ſol la*) ſhould de divided into three parts, ſo as to anſwer (pretty near) the Ariſtoxenians *Two Tones and an Half*; and might, altogether, make up the proportion of 4 to 3; which is that of a *Fourth* or *Dia-teſſaron*.

Many attempts were made to this purpoſe: And, according to thoſe, they gave Names to the different *Genera* or Kinds of Muſick, (the *Diatonick*, *Chromatick*, and *Enarmonick* Kinds,) with the ſeveral *Species*, or leſſer Diſtinctions under thoſe Generals. All which to enumerate, would be too large, and not neceſſary to our buſineſs.

The firſt was that of *Euclide* (which did moſt generally obtain for many ages :) Which allows to *Fa ſol*, and to *Sol la*, the full proportion of 9 to 8; And therefore to *Fa ſol la* (which we call the *greater Third*,) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$.) And, conſequently, to that of *Mi fa* (which is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64})\frac{3}{4}(\frac{256}{243}$; that is, if out of the proportion of 4 to 3, we take that of 81 to 64, the Reſult is that of 256 to 243. To this they gave the name of *Limma* (λεῖμμα) that is, the *Remainder* (to wit, over and above *Two Tones.*) But, in common diſcourſe (when we do not pretend to ſpeak nicely, nor intend to be ſo underſtood) it is uſual to call it an *Hemitone* or *Half-Note* (as being very near it) and, the other, two *Whole-Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with *Two full Tones.*)

Againſt this, it is objected (as not the moſt convenient Diviſion,) that the Numbers of 81 to 64, are too great for that of a *Ditone* or *Greater Third*: Which is not *Harſh* to the Ear; but is rather *Sweeter* than that of a ſingle *Tone*, who's proportion is 9 to 8. And in that of 256 to 243, the Numbers are yet much greater. Whereas there are many proportions (as $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$,) in ſmaller numbers than that of 9 to 8; of which, in this diviſion, there is no notice taken.

To rectify this, there is another Diviſion thought more convenient; which is *Ptolemy's Diatonum Intenſum* (of the *Diatonick* Kind, more *Intenſe* or Acute than that other..) Which, inſtead of Two *Full tones* for *Fa ſol la*; aſſignes (what we now call) a *Greater* and a *Leſſer* Tone; (which, by the more nice Muſicians of this and the laſt Age, ſeems to be more embraced;) Aſſigning to *Fa ſol*, that of 9 to 8 (which they call the *Greater Tone*:) and to *Sol la*, that of 10 to 9, (which they call the *Leſſer Tone*:) And therefore to *Fa la* (the *Ditone* or *Greater Third*) that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And conſequently, to *Mi fa* (which is remaining of the *Fourth*) that of 16 to 15. For $\frac{5}{4})\frac{4}{3}(\frac{16}{15}$. That is; if out of that of 4 to 3, we take that of 5 to 4, there remains that of 16 to 15.

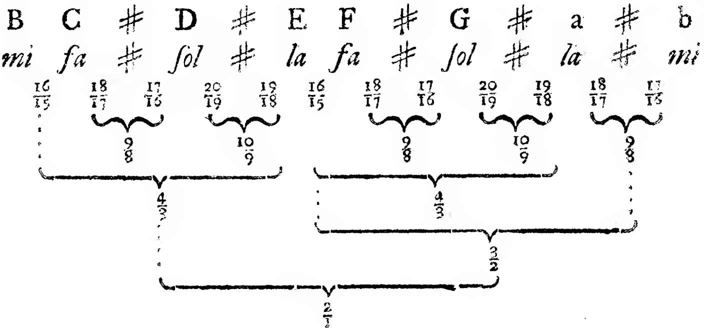
Many other waies there are (with which I ſhall not trouble you at preſent) of dividing the *Fourth* or *Dia-teſſaron*, or the proportion of 4 to 3, into three parts, anſwering to what (in a looſer way of Expreſſion) we call an *Half-note*, and two *Whole-notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as the moſt proper. To which therefore I ſhall apply my diſcourſe. Where $\frac{16}{15}$ is (what we call)the *Hemitone* or *Half-note*,in *Mi fa*; $\frac{9}{8}$ that of the *Greater-Tone*,in *Fa ſol*;and $\frac{10}{9}$ the *Leſſer-Tone*,in *Sol la*.

Onely with this addition; That each of thoſe Tones, is (upon occaſion) by *Flats* and *Sharps* (as we now ſpeak) divided into two *Hemitones* or *Half-notes*: Which anſwers to what by the Greeks was called *Mutatio quoad Modos* (the change of Mood ;) and what is now done by removing *Mi* to another Key. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus,

Thus, by the help of *Flats* and *Sharps* (dividing each *Whole-note*, be it the Greater or the Leſſer, into two *Half-notes*, or what we call ſo,) the whole *Octave* is divided into Twelve Parts or Intervals (contained between Thirteen Pipes) which are commonly called *Hemitones* or *Half-notes*. Not, that each is preciſely *Half a Note*, but ſomewhat near it, and ſo called. And I ſay, by *Flats and Sharps*; For ſometime the one, ſometime the other, is uſed. As, for inſtance, a *Flat* in D, or a *Sharp* in C, do either of them denote a *Midling* Sound (tho' not preciſely in the Midſt) between C and D; Sharper than C, and Flatter than D

Accordingly; ſuppoſing *Mi* to ſtand in B *fa* b *mi* (which is accounted its Natural ſeat) the Sounds of each Pipe are to bear theſe proportions to each other, viz.



And ſo in each Octave ſucceſſively following. And if the Pipes in each Octave be fitted to ſounds in theſe proportions of Gravity & Acuteneſs; it will be ſuppoſed (according to this Hypotheſis) to be perfectly proportioned.

But, inſtead of theſe ſucceſſive proportions for each Hemitone; it is found neceſſary (if I do not miſtake the practiſe) ſo to order the 13 Pipes (containing 12 Intervals which they call Hemitones) as that their Sounds (as to Gravity & Acuteneſs) be in Continual Proportion, (each to its next following, in one and the ſame Proportion;) which, all together, ſhall compleat that of an Octave or Dia-paſon, as 2 to 1. Whereby it comes to paſs, that each Pipe doth not expreſs its proper Sound, but very near it, yet ſomewhat varying from it; Which they call *Bearing*. Which is ſomewhat of Imperfection in this Noble Inſtrument, the Top of all.

It may be asked, Why may not the Pipes be ſo ordered, as to have their Sounds in juſt Proportion, as well as thus *Bearing* ?

I anſwer, It might very well be ſo, if all Muſick were Compoſed to the ſame *Key*, or (as the Greeks call it) the ſame *Mode*. As, for inſtance, if, in all Compoſitions, *Mi* were alwaies placed in B *fa* b *mi*. For then the Pipes might be ordered in ſuch proportions as I have now deſigned.

But Muſical Compoſitions are made in great variety of *Modes*, or with great diverſity in the *Pitch*. *Mi* is not always placed in B *fa* b *mi*; but ſometimes in E *la mi*; ſometimes in A *la mi re*, &c. And (in ſumme) there is none of theſe 12 or 13 Pipes but may be made the Seat of *Mi*. And if they were exactly fitted to any one of theſe caſes, they would be quite out of order for all the reſt.

As, for inſtance; If *Mi* be removed from B *fa* b *mi* (by a *Flat* in B) to E *la mi*: Inſtead of the Proportions but now deſigned, they muſt be thus ordered;

B		♯		C		♯		D		♯		E		F		♯		G		♯		a		b
fa		♯		*ſol*		♯		*la*		♯		*mi*		*fa*		♯		*ſol*		♯		*la*		*fa*
	18/17		17/16		20/19		19/18		18/17		17/16		16/15		18/17		17/16		20/19		19/18		16/15	

Where 'tis manifeſt, that the removal of *mi* doth quite diſorder the whole ſeries of Proportions. And the ſame would again happen, if *mi* be removed from E to A (by another *Flat* in E.) And again if removed from A to D. And ſo perpetually.

But the Hemitones being made all Equal; they do indifferently anſwer all the poſitions of *Mi* (though not exactly to any :) Yet nearer to ſome than to others. Whence it is, that the ſame Tune ſounds better at one Key than at another.

It is asked, Whether this may not be remedied; by interpoſing more Pipes; and thereby dividing a Note, not only (as now) into *Half-notes*, but into *Quarter-notes* or *Half-quarter-notes*, &c.

I anſwer; It may be thus remedied in part; (that is, the Imperfection might thus be ſomewhat Leſs, and the Sounds ſomewhat nearer to the juſt Proportions :) but it can never be exactly true, ſo long as their Sounds (be they never ſo many) be in continual proportion ; that is, each to the next ſubſequent in-the ſame Proportion.

For it hath been long ſince Demonſtrated, that there is no ſuch thing as a *juſt Hemitone* practicable in Muſick, (and the like for the diviſion of a Tone into any number of *Equal* parts; three, four, or more.) For, ſuppoſing the Proportion of a *Tone* or *Full-note*, to be $\frac{9}{8}$ (or, as 9 to 8 ;) that of the *Half-note* muſt be as $\sqrt{9}$ to $\sqrt{8}$ (as the *Square-root* of 9 to the *Square-root* of 8 ; that is, as 3 to $\sqrt{8}$, or 3 to $2\sqrt{2}$,) which are *Incommenſurable* quantities. And that of a *Quarter-note*, as $\sqrt{qq}\,9$ to $\sqrt{qq}\,8$, (as the *Biquadrate* root of 9, to the *Biquadrate* root of 8 ,) which is yet more *Incommenſurate*. And the like for any other number of Equal parts. Which will therefore never fall-in with the Proportions of Number to Number.

So that this can never be perfectly adjuſted for all Keys (without ſomewhat of Bearing) by multiplying of Pipes ; unleſs we would for every Key (or every different Seat of *Mi*) have a different Set of Pipes, of which this or that is to be uſed, according as (in the Compoſition) *Mi* is ſuppoſed to ſtand in this or that Seat. Which vaſt number of Pipes (for every Octave) would vaſtly increaſe the Charge. And (when all is done) make the whole impracticable.

Theſe are my preſent thoughts, of the Queſtion propoſed to me, and upon theſe grounds.

You will pleaſe to excuſe me for the trouble I give you of ſo long a Letter.

I thought it neceſſary, to give a little intimation of the Ancient Greek Muſick compared with what is now in practiſe ; which is more the ſame than moſt men are aware of : though the Language be very different. But I was not to be large in it. Thoſe who deſire to know more of it ; may ſee my thoughts more at large, in that Appendix which I have added at the end of my Edition of Ptolemy's Harmonicks in Greek and Latin.

The two Eminent Sects amongſt them,the *Ariſtoxenian* and the *Pythagorian*, differ much at the ſame rate as doth the Language of our ordinary practical Muſicians, and that of thoſe who treat of it in a more Speculative way.

Our Practical Muſicians talk of *Notes* and *Half-notes*, juſt as the *Ariſtoxenians* did ; as if the *Whole Notes* were all Equal ; and the *Half-notes* likewiſe each the juſt Half of a Whole Note.

And thus it is neceſſary to ſuppoſe in the Pipes of an Organ; which have each their determinate Sound and not to be corrected, in their little Inequalities, as the Voice may be by the guidance of the Ear.

But *Pythagoras*, and thoſe who follow him found (by the Ear) that this Equality of Intervals would not exactly anſwer the Muſical Appearances, in Concords and Diſcords: juſt as our Organiſts and Organ-makers be now aware; that their Pipes at equal Intervals do not give the juſt deſired Harmony, without ſomewhat of *Bearing*, that is, of ſome little variation from the juſt Sound.

The *Pythagorians*, to help this, changed the notion of *Equal Intervals* into that of *due Proportions*. And this is followed by *Zarline*, *Keppler*, *Cartes*, and others who treat of Speculative Muſick in this and the laſt Age. And though they ſpeak of Notes and Half-notes (in a more groſs way) much as others do, yet declare themſelves to be underſtood more nicely.

And though our preſent *Gam-ut* take no notice of this little diverſity; yet, in Vocal Muſick, the Ear directs the Voice to a more juſt proportion. And, in String Muſick, it may in like manner be helped by ſtraining and ſlackening the Strings, or moving the Frets. But, in Wind Muſick, the Pipes are not capable of ſuch correction ; and therefore we muſt be content with ſome little irregularity therein; that ſo they may tolerably anſwer (though not exactly) the different Compoſitions according to the different placing of *Mi* in the *Gam-ut*.

Now the Deſign of Mr. *Harris* ſeems to be this; either (by multiplying intermediate Pipes) to bring the Organ to a juſt Perfection : Or elſe (if that cannot be done) to reſt content with the little Imperfection that is ; which though, by more Pipes, it may be ſomewhat abated, yet cannot be perfectly remedied. And in this I think we muſt acquieſce.

I am

SIR,

Yours to ſerve you

Oxford June
27. 1698.

JOHN WALLIS.